a certain influence on the intensity of disturbances formed in the supersonic jet under sound excitation is a result of the relation between the oscillating velocity vector direction in the incident sound wave and the jet issue velocity direction.

Conclusion

Thus, at certain incidence angles of the sawtooth-like sound waves of a finite amplitude to the linearly unstable shear layer of a supersonic jet, a significant amplification of sound radiated by such a jet at the external excitation frequency is possible. This radiation is connected with the intensity increase of disturbances generated in the jet under sound excitation and, as a consequence, with the intensity increase of Mach waves radiated by them.

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Numerical Stability Conditions in the Calculation of Potential Velocity Fields

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Introduction

IN recent years, the development of numerical calculation has lead to reductions in calculation times and costs and has enabled a deeper understanding of the flow structure through turbomachinery and physical processes that govern the internal phenomena.

The singularities method is used to analyze the flow through turbomachinery.[1–5] This method enables particular solution of the Laplace equation, which satisfies the imposed boundary conditions. The method is very useful because it allows one, once the calculation programs are ready, to study the flows by the superposition of the following elementary flows[6–8]: a basic uniform flow and sources, sinks, or vortices located at well-chosen points in the flowfield. The aim of this study is to highlight the problems met during the numerical programming of the method and also to propose, in many cases, a solution. A large part of this work is based on the Joukowski profiles for which the analytical solution of the flow is known.[9] We adopt the following nomenclature to define these profiles: Jouko 04 80 10, where Jouko is the Joukowski profile, 04 is the absolute value of the camber angle β (deg), 80 is the [1 − relative thickness] × 100 ($e = 20\%$), and 10 is the angle of attack α_0 (deg).

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The Joukowski profile, with its fine or cusped trailing edge, presents the most difficulties for application of the Kutta condition and, consequently, for numerical stability. This profile is presented as an ideal means to validate a potential calculation code. A numerical study has been developed to discuss the accuracy of the solutions. The aims of this study are summarized as follows:

1) Provide an accurate solution for these different problems. For example, some profiles, obtained by the Joukowski transformation present, in spite of an analytical definition, a crossing of the suction and pressure sides at the trailing edge. This crossing causes a serious error in the velocity field computation. A new procedure to solve this problem is presented.

2) Expose the elements that influence the method: precision in the geometrical profile definition, trailing-edge geometry, smoothing problems, number of discretization points, precision of calculation, etc.

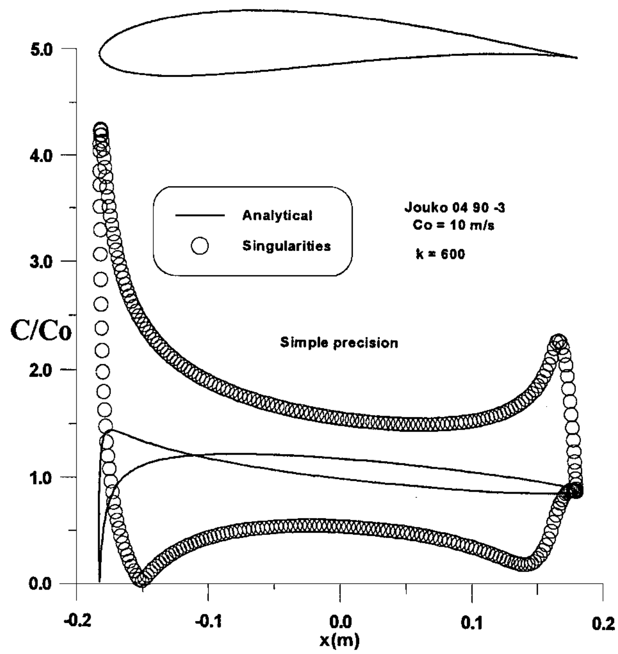

Fig. 1a Simple precision.

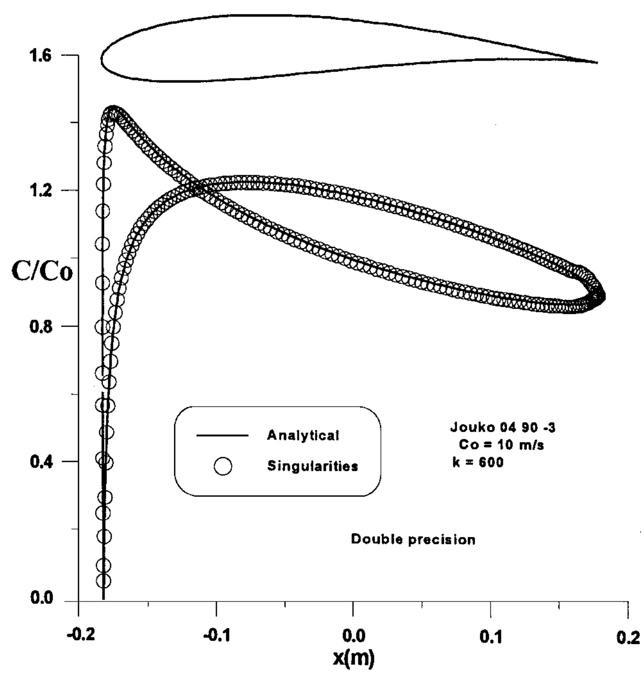

Fig. 1b Double precision.

Numerical Study

The flexibility of the method and accuracy of results depend mainly on the profile geometry (precision of the surface smoothing, trailing-edge geometry, etc.). Many authors have studied the numerical aspects of this method.[10–14] In the following section these aspects are presented and accurate solutions are provided for the different problems.

Precision and Method of Calculation

The choice of whether to conduct calculations using simple or double precision is of great importance for the quality of results.[15] Using the same principle, the velocity field, discretized into 600 segments around a Joukowski profile, was studied using simple and double precision. Figures 1a and 1b show the sensitivity of the method to the truncation error.

The considerable difference between the two sets of results can be explained by considering the structure of the matrix A_{ij}. Circulations

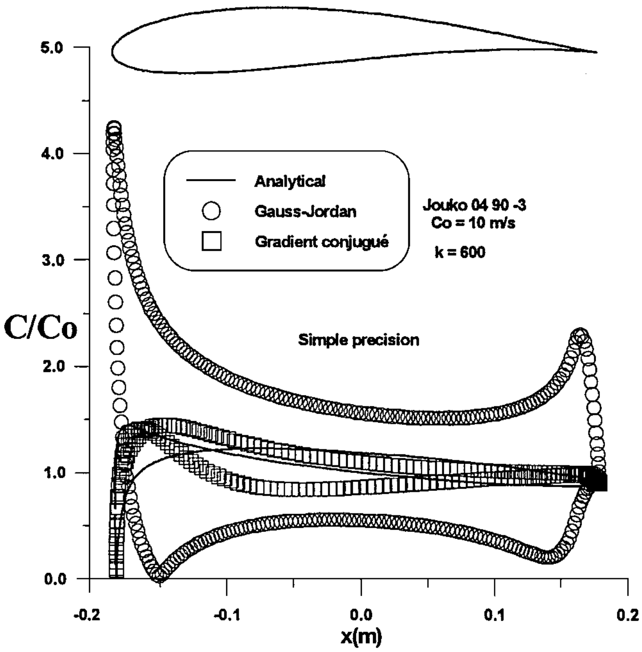

Fig. 2a Simple precision.

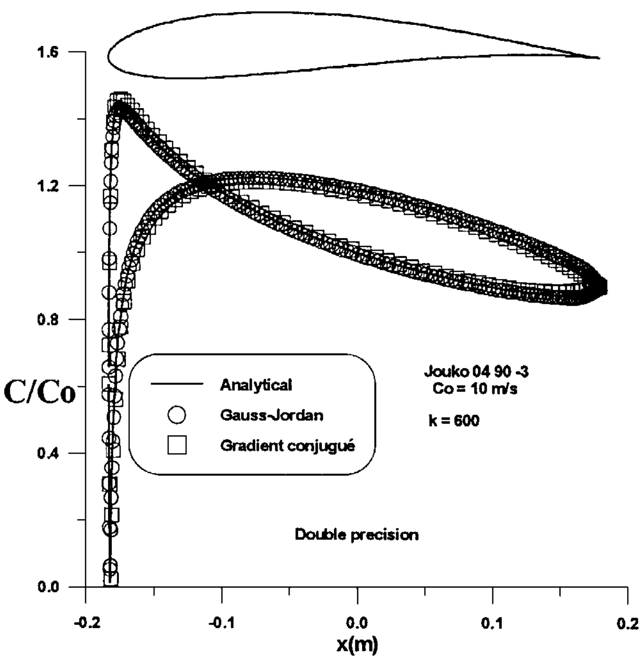

Fig. 2b Double precision.

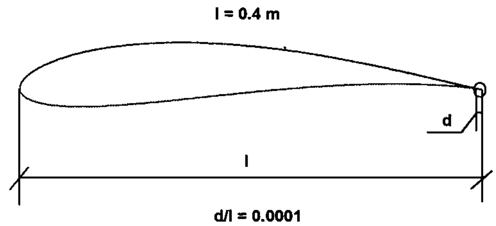

Fig. 3a Joukowski profile global view.

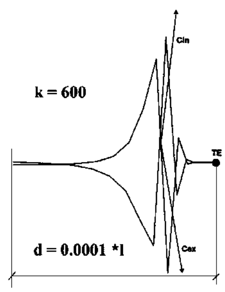

Fig. 3b Joukowski profile-zoom at the trailing edge.

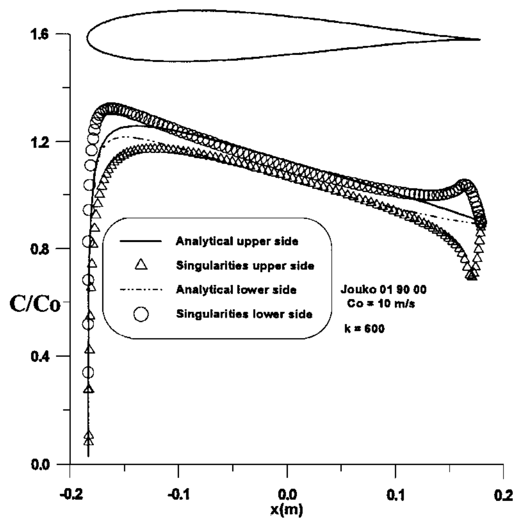

Fig. 3c Velocities field before smoothing the trailing edge.

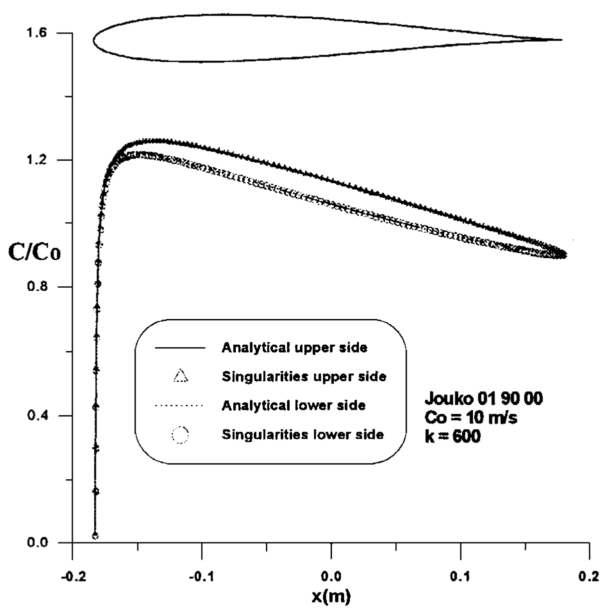

Fig. 3d Velocity field after smoothing the trailing edge.

Γ_j and, subsequently, velocities C_i are calculated by way of this matrix. It is necessary, in some cases, to have a well-conditioned matrix before calculating the velocity field. To highlight the influence of the method of calculation on the obtained result, the velocity field has been calculated around the Joukowski profile using two methods. The first, that of Gauss–Jordan, is direct, is more accurate, and uses less computing time. Computing time using a direct solution is proportional to k^3. This time is independent of the number of calculated right sides. The second method is the iterative conjugate gradient method for which the computing time for each right side is proportional to (iter $\times$ k^2), where iter is the number of iterations needed for convergence. In the case of simple precision where the matrix is unstable (Fig. 2a), the iterative method is quite clearly more accurate than the direct method. For double precision (Fig. 2b), however, it is preferable to use the direct method because it requires less computing time, around about 1 min on a Pentium personal computer, 166 MHz with 32 megaoctet (Mo) of RAM.

Geometrical Defect at the Trailing Edge Related to the Conformal Mapping

For the Joukowski profiles, in spite of double-precision calculation for 600 discretization points, there is a crossing of the suction and pressure sides at the trailing edge. This instability, visible only with a large zoom, leads to a very serious error in the obtained field, even when crossing takes place at a very small distance d from the trailing edge, as shown in Figs. 3a–3c. To solve this problem, smoothing is used at the trailing edge. Once this is done, the Kutta condition is employed at the last segments at the trailing edge. A very acceptable velocity field has been obtained (Fig. 3d). Table 1 confirms the result.

Trailing-Edge Geometry of a Thin Profile

The influence of a thinner trailing edge on calculation results has been the subject of many studies.[16,17] For our part, we have shown that despite trailing-edge smoothing, some irregularities still exist, and furthermore, they are linked to profile geometry accuracy. The discretization of the latter induces circulation "Γ" instability

Table 1 Comparison of total calculated and theoretical circulation

Γ_{th}	0.2193
Γ_{total} (before smoothing the trailing edge)[a]	−0.8105
Γ_{total} (after smoothing the trailing edge)[a]	0.2192

[a] Where

$$\Gamma_{total} = \sum_{j=1}^{k} \Gamma_j$$

where Γ_j are the intensity of the vortices distributed at the profile surface.

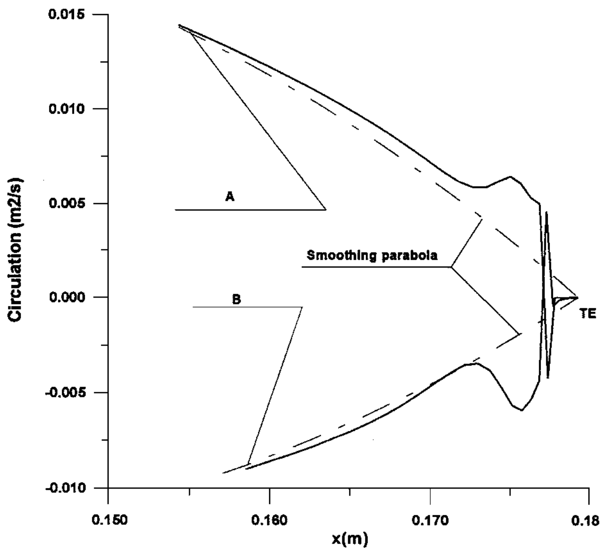

Fig. 4a Circulation at the trailing edge.

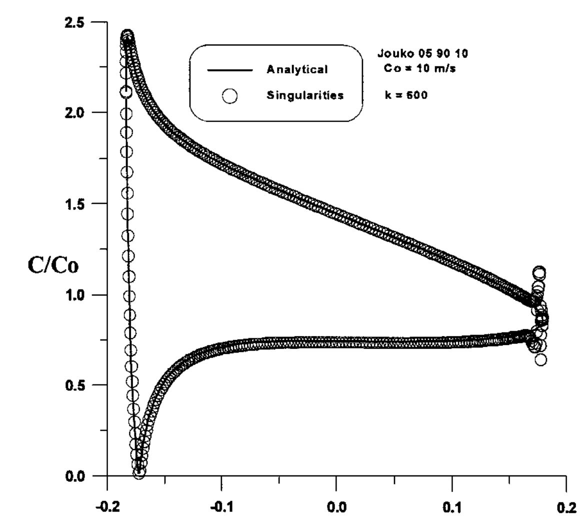

Fig. 4b Velocity field on the surface of the Joukowski profile.

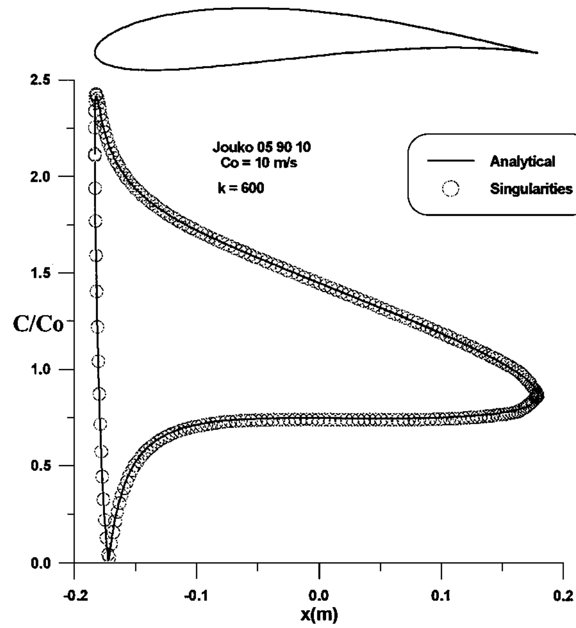

Fig. 5a Velocity field.

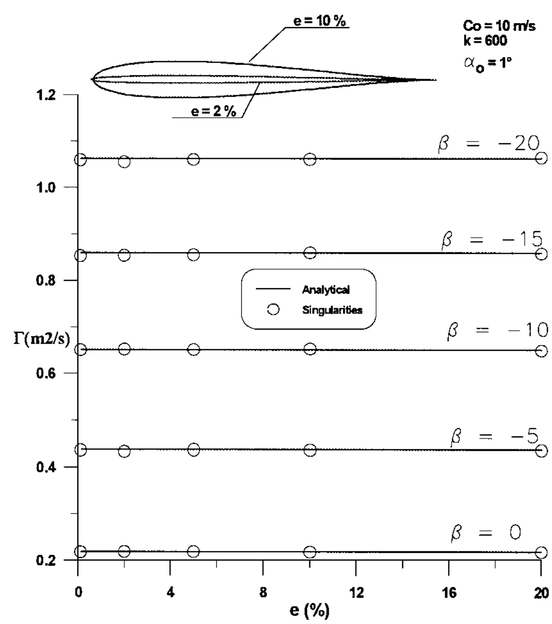

Fig. 5b Comparison of circulation for varying relative thicknesses and cambers.

(Fig. 4a) leading to velocity fluctuation (Fig. 4b). Indeed, the more the profile is discretized, the smaller the last two segments become, tending toward zero and, thus, rendering the influence matrix more unstable (no strong main diagonal). To solve this problem, Girardi and Bizarro[18] modified the method developed by Hess and Smith[7] by using a weighting function to define the circulation around the Joukowski profile and over a plane cascade. Here a circulation weighting function has been adopted, as defined in Fig. 4a. Upstream of the fluctuation (at points A and B located at 95% of the chord) and until the trailing edge (point TE), the circulation is smoothed by the following parabolic function:

$$\Gamma(x) = a \cdot x^2 + b \cdot x + c$$

where a, b, and c are constants defined according to the following conditions: The circulation is zero at the trailing edge in accordance with the Kutta condition (one equation). Upstream of the fluctuation, joining is carried out by a common tangent (two equations).

After smoothing the circulation, a very satisfactory result for total circulation with a relative error less than 0.5% is obtained (Fig. 5a). Figure 5b shows one comparison between calculated and theoretical circulation for varying relative thickness $e\%$ and camber angle β.

In this application and with circulation smoothing, the problems cited by several authors[19,20] concerning the low relative thicknesses are unfounded.

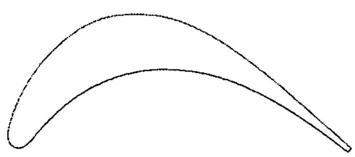

Fig. 6a Turbine profile.

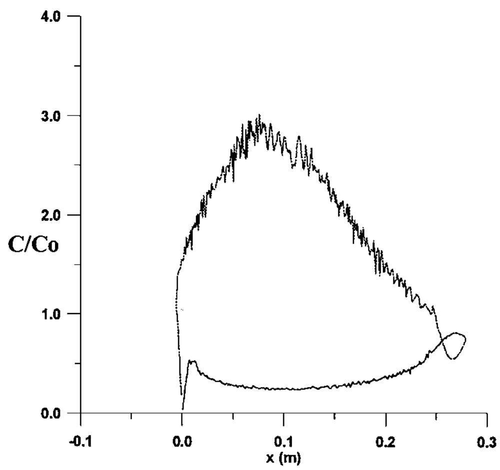

Fig. 6b Velocity field.

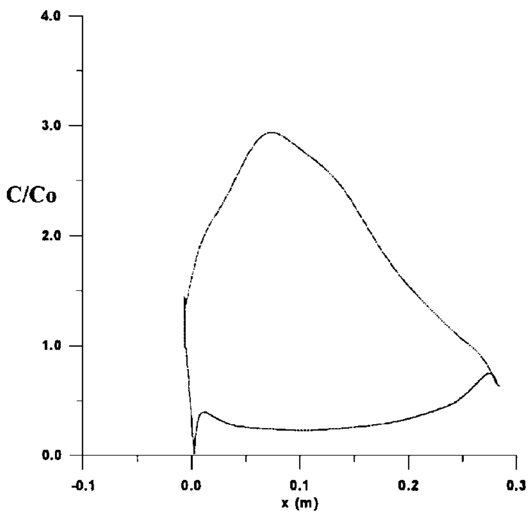

Fig. 6c After smoothing.

Roughness of the Profile Surface

The linear system matrix elements depend directly on the profile geometry.[21] To illustrate this, Figs. 6a and 6b show the influence of the surface roughness on the velocity field, calculated at the surface of a turbine blade, discretized into 600 segments. A groove on the profile surface, practically invisible to the naked eye, induces velocity fluctuations and a strong local velocity gradient creating, during boundary-layer calculations, unrealistic flow separation.

To solve this problem, smoothing of the profile is adopted. Generally, the smoothing comprises two functions defining the suction and the pressure sides of the profile, respectively, joined by two circles at the leading and trailing edges. The smoothing, using two polynomial functions whose power depends on the minimization of the standard deviation between the exact and interpolated values, gives a regular velocity field (Fig. 6c). Here a polynomial of 8th power is used on the suction side and a polynomial of 10th power on the pressure side.

Conclusion

This study has highlighted the suitability of the singularities method for calculation of the flow around a profile, in comparison with analytical results, as well as its sensitivity to various numerical parameters. Of utmost importance is the accuracy with which the profile geometry is defined. Smoothing the surface is essential for profiles defined by experimental data; this being generally the case. An after treatment of the trailing-edge geometry, based on current visualization technology, is required to correctly apply the Kutta condition. For the general case and during the study of the flow over a profile using the singularities method, the following procedure is recommended to gain accurate solutions:

1) Study the trailing-edge geometry. If this indicates instability (crossing of the suction and pressure sides), smoothing is necessary.

2) After having smoothed the trailing edge, circulation smoothing is essential in certain cases, e.g., for profiles with a cusped trailing edge.

3) Recent compilers, using the RAM, have allowed vast increases in the number of discretization points, giving even more accurate solutions.

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Experimental Buckling of Thin Composite Cylinders in Compression

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Introduction

THE thin, circular, cylindrical shell has been used extensively as a structural configuration, mainly in the aerospace industry. The specific concern of its behavior when subjected to external loads and in particular the buckling phenomena has, therefore, received significant attention.[1] Recently, the increasing need for lightweight efficient structures has driven research in the field of structural optimization and simultaneously to the use of fiber-reinforced composites, which are attractive because of their high stiffness-to-weight ratios. Many studies have been conducted on the buckling of composites; however, these studies have not yet led to systematic and widely applicable design criteria. This is primarily due to the multitude of parameters that influence the instability but also to the lack of available representative tests results. The buckling of composite shells, in addition to the initial geometric imperfections, also typical for isotropic shells, depends on a large number of input parameters, such as lamina properties and orientations, and can be influenced by several types of imperfections, consequences of the manufacturing process, such as thickness variations[2] and local delaminations.[3] To develop appropriate methods for predicting the buckling loads of thin shells, it is important to increase the available data on the shape and amplitudes of initial imperfections,[4] as well as the results of experimental buckling tests on composite shells of different materials and layup orientations.[5,6] In fact, the experimental results have proven to be extremely useful for tuning analytical[7] and numerical[8,9] models.

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This Note describes the experimental equipment and the methodologies used for performing buckling tests under position control on composite cylindrical shells subject to axial compression, measuring the development of the geometric imperfections and of the buckling pattern. The results of tests on 16 thin shells in carbon fabric, carbon unidirectional laid down, and carbon roving tape wrapped with different layup orientations are reported and evaluated. Typical results, showing variations of compressive load with axial displacement and postbuckling patterns, are presented to demonstrate the reliability and accuracy of the experimental setup.

Cylindrical Shells

The cylindrical specimens are characterized by a length and an internal diameter of 700 mm. Two tabs are provided at the top and bottom surfaces for attaching them to the loading rig. The actual length is, therefore, limited to the central part and is equal to 540 mm.

Testing has been conducted on 16 specimens, made of carbon fabric, of carbon unidirectional laid down, and of carbon roving tape wrapped in epoxy matrix (Table 1). They are of different staking sequences and have a total thickness equal to 1.32, 1.20, and 1.50 mm for the carbon fabric, the carbon unidirectional, and the carbon roving tape wrapped specimens, respectively. The lamina properties of the carbon fabric cylinders are $E_{11} = E_{22} = 52{,}000\ \text{N/mm}^2$,

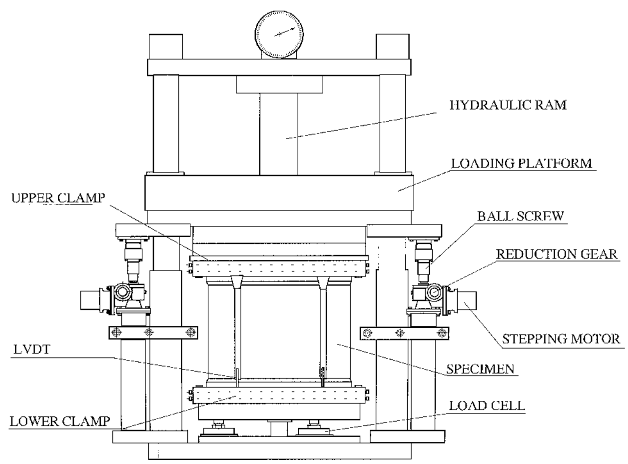

Fig. 1a Loading rig to perform axial compression buckling tests under position control.

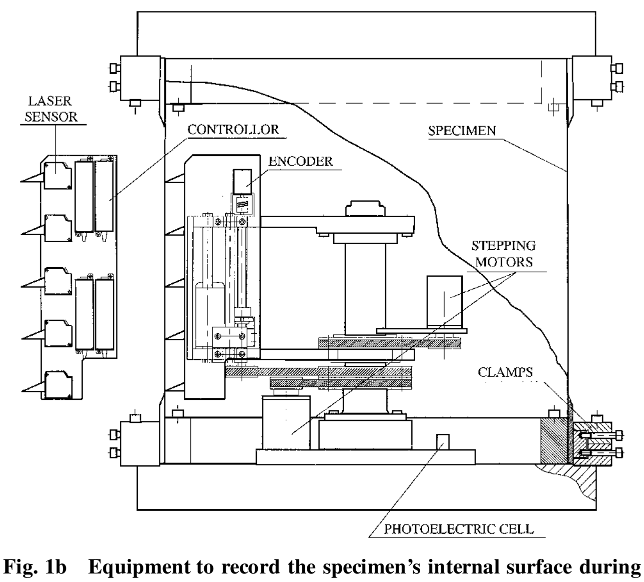

Fig. 1b Equipment to record the specimen's internal surface during the tests.